Nuclear symmetry energy from QCD sum rules

JEONG Kie Sang* LEE Su Houng

Institute of Physics and Applied Physics, Yonsei University, Seoul 120-749, Korea

Abstract We calculated the nucleon self-energies in iso-spin asymmetric nuclear matter and obtained the nuclear symmetry energy by taking difference of these of neutron and proton. We find that the scalar (vector) self-energy part gives a negative (positive) contribution to the nuclear symmetry energy, consistent with the result from relativistic mean-field theories. Also, we found exact four-quark operator product expansion for nucleon sum rule. Among them, twist-4 matrix elements which can be extracted from deep inelastic scattering experiment constitute an essential part in the origin of the nuclear symmetry energy from QCD. Our result also extends early success of QCD sum rule in the symmetric nuclear matter to the asymmetric nuclear matter.

Key words Nuclear symmetry energy, Relativistic mean field theory, QCD sum rule, Twist-4 matrix elements

1 Introduction

Recently the Korea government decided to build low energy rare isotope accelerator, RAON, which will be mainly used to study exotic properties of iso-spin asymmetric nuclear matter ranging from rare isotope to neutron star. To understand the exotic property of the asymmetric nuclear matter, one should know what cause the difference with the symmetric nuclear matter.

In Dirac phenomenology, strong vector repulsion and scalar attraction in optical potential is needed to explain the phenomena in nucleon-nucleus scattering and these tendency naturally comes from successive relativistic mean field type model calculations.

Almost 20 years ago, this tendency had been confirmed in QCD degrees of freedom, using finite density QCD sum rule, by Cohen, Furnstahl and Griegel[1-4]. These studies show that nucleon bulk properties can be understood in QCD degree of freedom.

With these two motivations and in a hope to express the origin of the nuclear symmetry energy in QCD degree of freedom, we have applied QCD sum rule to the asymmetric nuclear matter.

2 Brief description for theoretical method

2.1 Phenomenological approach

There was early attempt which describe iso-spin asymmetry in a finite nuclei by taking linear potential. Following is well known Bethe-Weizsäker formula:

$$m_{\mathrm{tot}} = Nm_n + Zm_p - \frac{E_B}{c^2},$$

$$\begin{aligned} E_B = {} & a_V A - a_S A^{\frac{2}{3}} - a_C\left[Z(Z-1)\right]A^{-\frac{1}{3}} \\ & - a_A I^2 A + \delta(A,Z), \end{aligned} \tag{1}$$

where $I=(N-Z)/A$, the fourth term in E_B accounts for the total shifted energy due to neutron number excess. When the nuclear symmetry energy is assumed to depend on linear density term, the fourth term can be easily calculated as

$$a_A = \frac{1}{4I}\left[E_n(\rho,I) - E_p(\rho,I)\right]. \tag{2}$$

For infinite nuclear matter case, energy density per nucleon of asymmetric (neutron rich) nuclear matter can be written as follows:

$$E = \overline{E_n}\rho_n + \overline{E_p}\rho_p = E(\rho) + E_{\mathrm{sym}}(\rho)I^2 + \cdots, \tag{3}$$

Supported by Korea national researchfoundation (Grants Nos.KRF-2011-0030621 and KRF-2011-0020333)

* Corresponding author. *E-mail address:*key.s.jeong@gmail.com

Received date: 2013-06-27

where $\overline{E_n}$ ($\overline{E_p}$) is the averaged quasi-nucleon energy in asymmetric nuclear matter which can be obtained by density integration of single nucleon energy E_n. If one assume linear density dependence of E_n (E_p), it is easy to get the averaged single nucleon energy.

By mean-field type approximation which gives plausible and clear view for the single nucleon energy in the nuclear matter, a single nucleon in the asymmetric nuclear matter can be regarded as a quasi-nucleon on the asymmetric Fermi-sea with large self-energies. From the difference of each nucleon self-energy, the nucleon symmetry energy can be obtained.

2.2 QCD sum rule for nucleon in asymmetric nuclear matter

In this study, we followed the formalism in Ref.[1-4] and detailed descriptions such as equations and formulas are given in Ref.[5]. The brief steps are as follows.

1) QCD sum rule starts with calculating the correlation function. The correlation function contains all possible quantum states of quasi-nucleon in the asymmetric nuclear matter.
2) The correlation function can be directly calculated by operator product expansion (OPE). The OPE of correlation function can be written as multiples of Wilson coefficient and localized operator expectation value, the condensate. At short distance, the Wilson coefficient can be calculated in perturbative diagram calculation.
3) Also, the correlation function is expected to have spectral structure similar as the propagator in relativistic mean field theory (RMFT). In this phenomenological ansatz, the nucleon self-energies are located near the quasi-nucleon pole state.
4) Because we are only interested in the self-energies which live near the quasi-nucleon pole, the OPEs and phenomenological side should be weighted. After the weighting process named as Borel transformation, the pole contribution in both expression becomes enhanced and the continuum contribution becomes suppressed.
5) By equating both weighted expression, one can relate the nucleon self-energies with OPE terms, in QCD degrees of freedom.

3 Nuclear symmetry energy from QCD sum rules

To check the physical quantity calculated from QCD sum rule, one should set appropriate range of Borel mass. It requires following criteria:

1) The contribution of highest mass dimension condensate should not excess 50% of the contribution of lowest mass dimension condensate.
2) The continuum contribution of each OPE term should not excess 50% of the pole contribution.

The nuclear symmetry energy which contains the condensate up to mass dimension-five is shown in Fig.1.

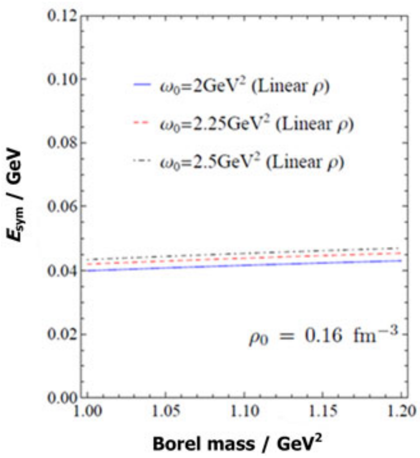

Fig.1 (Color online) Nuclear symmetry energy which contains up to mass dimension-five condensates.

From Fig.1, one can check that the nuclear symmetry energy has a value near 40 MeV, consistent with previous studies in order of magnitude.

In following, we present the nuclear symmetry energy which contains up to mass dimension-six condensates. The section is divided into the two cases, one contains twist-4 matrix elements and the other does not.

So far, in nucleon OPE, no exact contribution of four-quark operators has been known. In this study we have found the exact four-quark operators. Especially, among them, dimension-six spin-2 operators (twist-4) can be extracted from DIS experiment[6] and their contribution to the nuclear symmetry energy is so important.

3.1 Results without twist-4 operator contribution

If one just account the mass dimension-six spin-1 condensates, the result is as following figure.

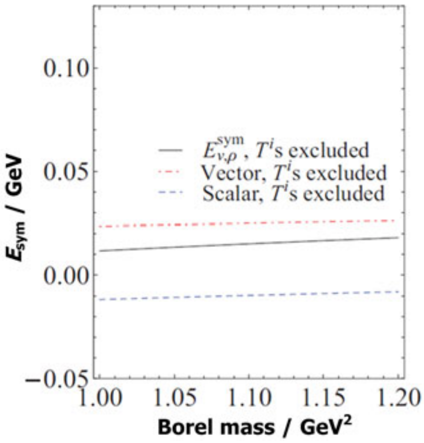

Fig.2. (Color online) Nuclear symmetry energy which contain up to mass dimension-six condensate (without twist-4). This result deals same result presented in Phys Rev C, 87: 015204.

From Fig.2, one can find the weak vector repulsion and scalar attraction consistent with the RMFT result which includes iso-vector meson exchange channel[7].

3.2 Results with twist-4 operator contribution

By including the twist-4 contribution, one can check that the mechanism of repulsion-attraction becomes more apparent one from Fig.3. This result shows that the reaction mechanism in the iso-spin asymmetric nuclear matter may be originated from multi-quark operators such as twist-4 operators.

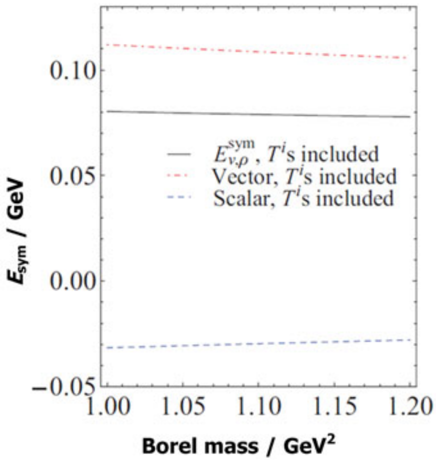

Fig.3 (Color online) Nuclear symmetry energy which contains up to mass dimension-six condensate with twist-4 matrix elements. This result deals same result presented in Phys Rev C, 87: 015204.

4 Conclusion

In this study, we calculated the nucleon self-energy in asymmetric nuclear matter in QCD degree of freedom by using finite density QCD sum rule technique. We successfully reproduced the nuclear symmetry energy of previous studies in order of magnitude and found the exact four quark condensates in the nucleon sum rule. From the calculation results, we confirmed meaningful fact that the iso-vector meson exchange mechanism suggested from many RMFT type studies may have origin from multi quark operators such as twist-4 operators. The contribution of twist-4 matrix element mimics following RMFT results[7].

$$\mathrm{E}_V^{\rm sym} = \frac{1}{2}\left[f_\rho - f_\delta \left(\frac{m^*}{E_F^*} \right) \right] \rho_B. \tag{4}$$

The twist-4 matrix elements contain some amounts of uncertainties, but the upgrade plan of Jefferson Lab. may provide precise twist-4 matrix elements and from that, we could get deeper understanding of the role of twist-4 operators in the nuclear symmetry energy. Still yet the high density behavior of the nuclear symmetry energy has not been uncovered. Such unknown properties can be studied by experimental study, for example by using RAON, or by theoretical study.

References

1 Cohen T D, Furnstahl R J, Griegel D K. Phys Rev Lett, 1991, 67: 961–964.

2 Furnstahl R J, Griegel D K, Cohen T D, Phys Rev C, 1992, 46: 1507–1527.

3 Jin X M, Cohen T D, Furnstahl R J, *et al.* Phys Rev C, 1993, 47: 2882–2900.

4 Jin X M, Nielsen M, Cohen T D, *et al.* Phys Rev C, 1994, 49: 464–477.

5 Jeong K S and Lee S H. Phys Rev C, 2013, 87: 015204.

6 Choi S, HatsudaT, Koike Y, *et al.* Phys Lett B, 1993, 312: 351.

7 Baran V, Colonna M, Greco V, *et al.* Phys. Rep, 410: 335–466.